GEOMETRÍA

PLANA DE GEOMETRÍA

CICLO REPASO UNI

ACADEMIA CÉSAR VALLEJO

PRÁCTICA N° 4

- ☐ ÁREAS DE REGIONES PLANAS II
- ☐ GEOMETRÍA DEL ESPACIO
- ☐ ÁNGULO POLIEDRO

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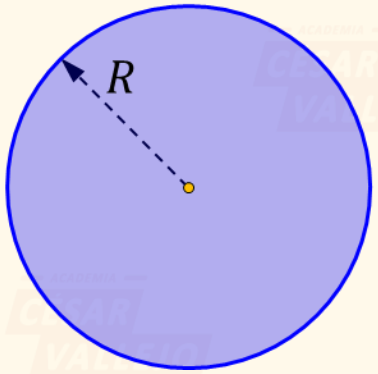
ÁREAS DE REGIONES PLANAS II

ÁREA DEL CÍRCULO



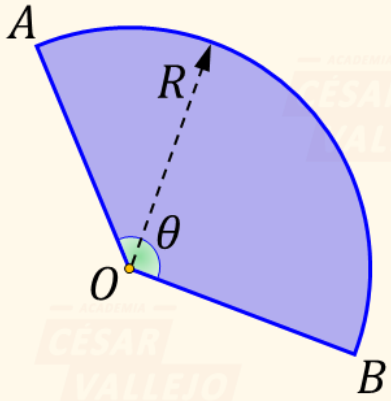
- En el gráfico, se cumple:

$$\mathbb{A}_{\bigcirc} = \pi \cdot R^2$$

Donde:

$\pi \approx 3{,}1415 \ldots$

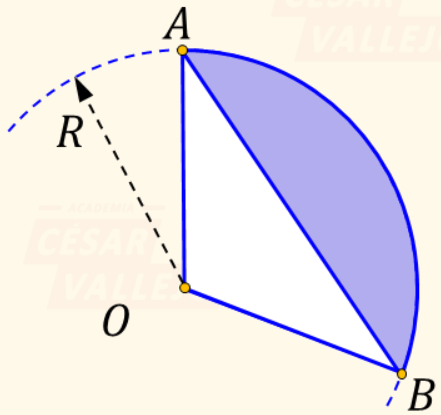
SECTOR CIRCULAR



Se cumple:

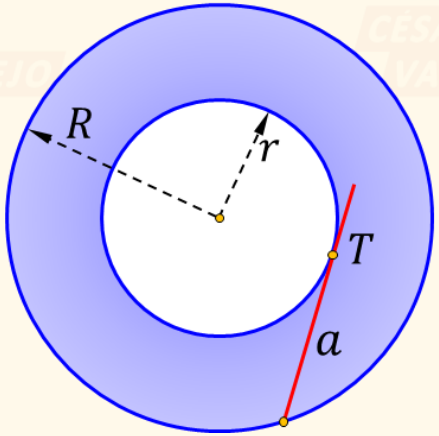
$$\mathbb{A}_{\triangle} = \frac{\pi \cdot R^2 \cdot \theta}{360°}$$

SEGMENTO CIRCULAR



$$\mathbb{A}_{)} = \mathbb{A}_{\triangle} - \mathbb{A}_{\triangle}$$

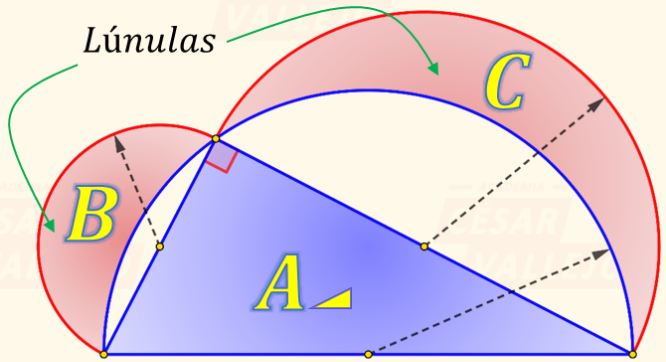
CORONA CIRCULAR



$$\mathbb{A}_{\circledcirc} = \pi(R^2 - r^2)$$

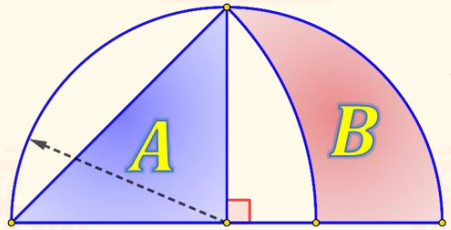
Si T es punto de tangencia.

$$\mathbb{A}_{\circledcirc} = \pi a^2$$



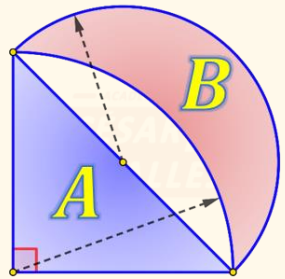
Se cumple:

$$A_{\triangle} = B + C$$



Se cumple:

$$A = B$$



Se cumple:

$$A = B$$

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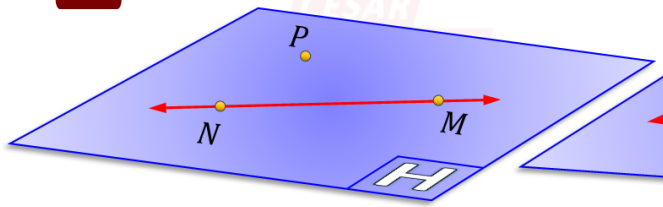
GEOMETRÍA DEL ESPACIO

DETERMINACIÓN DEL PLANO

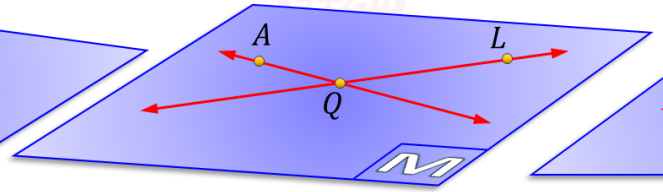
Una recta y un punto exterior a ella, determinan un plano al cual pertenecen.



- $P \notin \overleftrightarrow{NM}$

→ P y $\overleftrightarrow{NM}$ determinan al $\mathbb{H}$

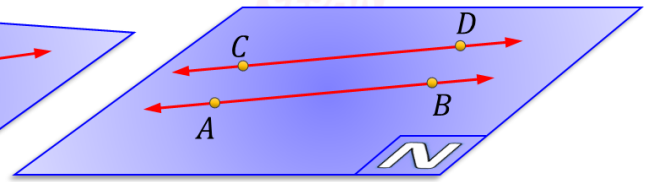
Dos rectas secantes, determinan un plano al cual pertenecen.



- $\overleftrightarrow{AQ} \cap \overleftrightarrow{QL} = \{Q\}$

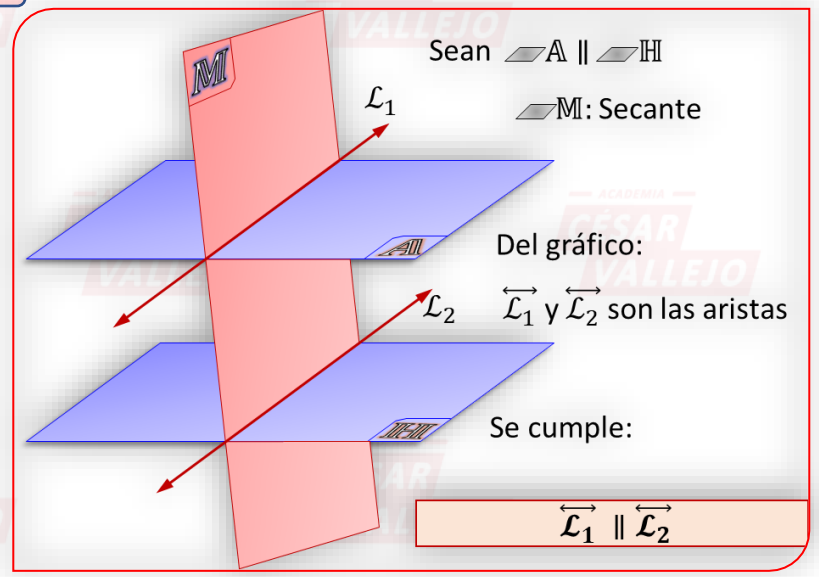
→ $\overleftrightarrow{AQ}$ y $\overleftrightarrow{QL}$ determinan al $\mathbb{M}$

Dos rectas paralelas, determinan un plano al cual pertenecen.



- $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

→ $\overleftrightarrow{AB}$ y $\overleftrightarrow{CD}$ determinan al $\mathbb{N}$



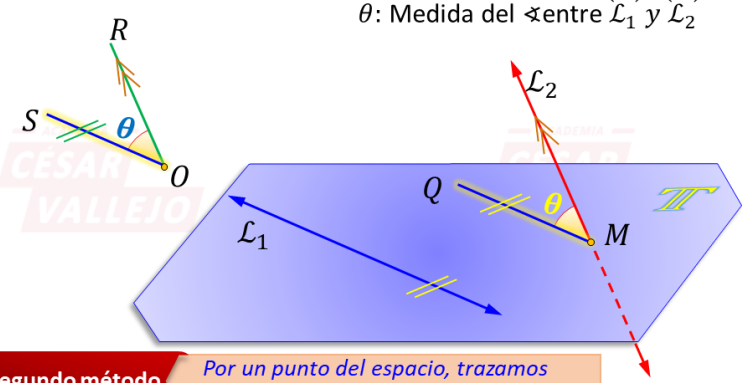
ÁNGULO ENTRE DOS RECTAS ALABEADAS

Sean $\overleftrightarrow{\mathcal{L}_1}$ y $\overleftrightarrow{\mathcal{L}_2}$ dos rectas alabeadas,

Primer método *Por un punto de una de las alabeadas, trazamos una paralela a la otra*

Por M trazamos: $\overline{MQ} \parallel \overleftrightarrow{\mathcal{L}_1}$

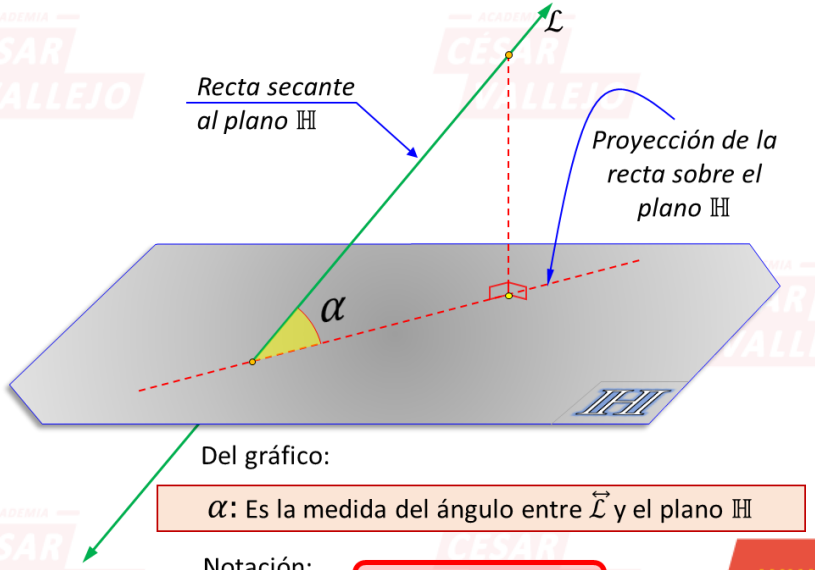
θ: Medida del $\measuredangle$ entre $\overleftrightarrow{\mathcal{L}_1}$ y $\overleftrightarrow{\mathcal{L}_2}$



Segundo método *Por un punto del espacio, trazamos paralelas a ambas rectas alabeadas*

Por O trazamos las paralelas a $\overleftrightarrow{\mathcal{L}_1}$ y $\overleftrightarrow{\mathcal{L}_2}$

$m\measuredangle ROS = \theta$: Medida del $\measuredangle$ entre $\overleftrightarrow{\mathcal{L}_1}$ y $\overleftrightarrow{\mathcal{L}_2}$

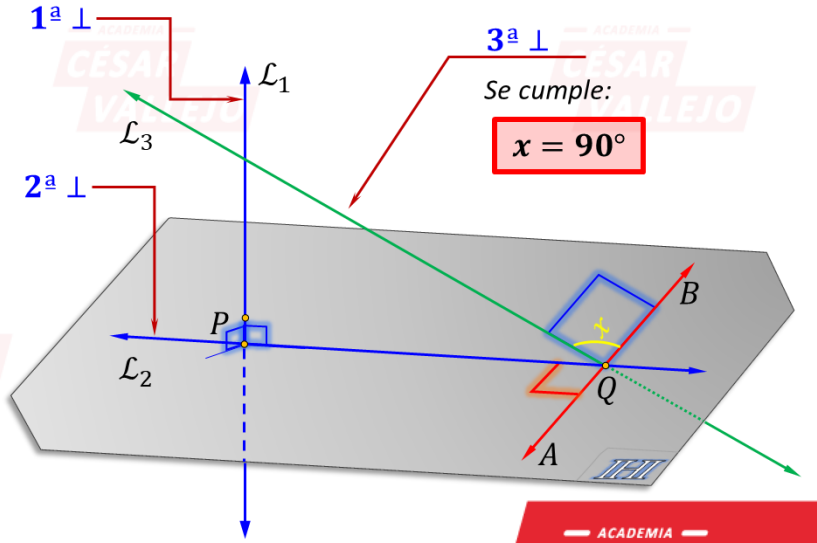


Del gráfico:

α: Es la medida del ángulo entre $\overleftrightarrow{\mathcal{L}}$ y el plano $\mathbb{H}$

Notación: $m\measuredangle\left(\overleftrightarrow{\mathcal{L}}; \mathbb{H}\right) = \alpha$

TEOREMA DE LAS TRES PERPENDICULARES



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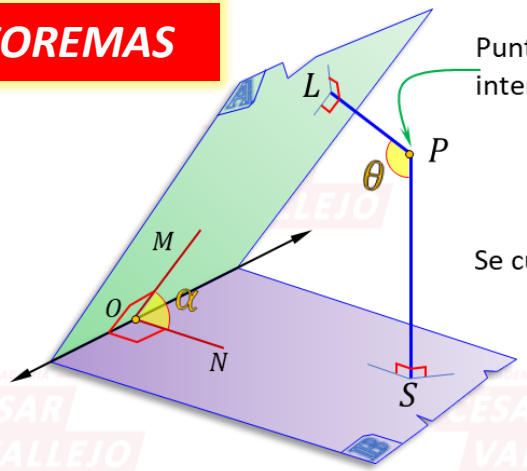
ÁNGULO POLIEDRO

TEOREMAS



Punto cualquiera en la región interior del diedro

$\overline{PL} \perp \mathbb{A}$

$\overline{PS} \perp \mathbb{B}$

Se cumple:

$$\alpha + \theta = 180°$$

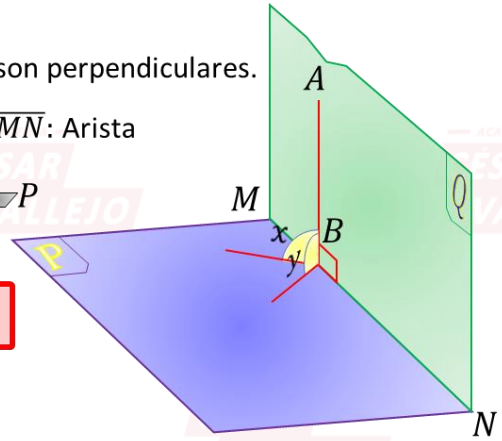
Los planos mostrados son perpendiculares.

Si $\overline{AB} \perp \overline{MN}$, donde $\overline{MN}$: Arista

Se cumple: $\overline{AB} \perp P$

Entonces:

$$x = y = 90°$$

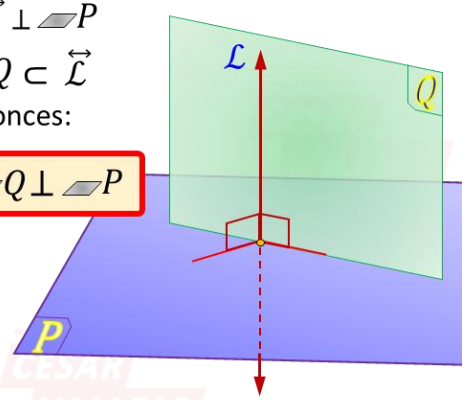


Si $\overleftrightarrow{\mathcal{L}} \perp P$

$Q \subset \overleftrightarrow{\mathcal{L}}$

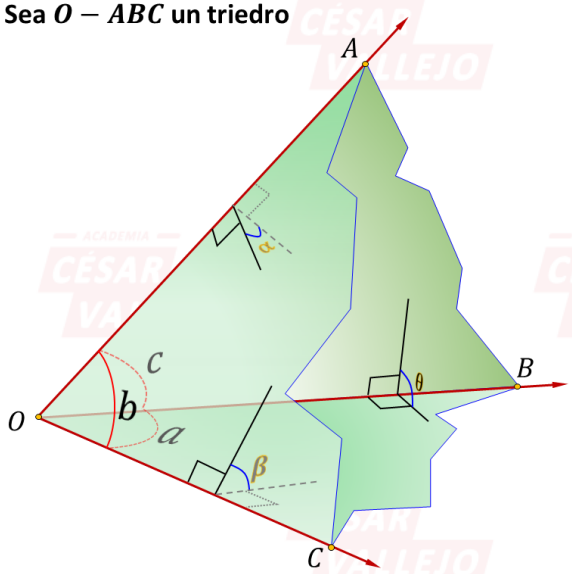
Entonces:

$Q \perp P$



TEOREMAS Generales

Sea $O - ABC$ un triedro



❑ Respecto a las caras

$0° < a + b + c < 360°$

$b - c < a < b + c$

❑ Respecto a los diedros

$180° < \alpha + \beta + \theta < 540°$

$\beta + \theta < \alpha + 180°$

❑ Teorema de correspondencia

Si $c < b$ $\Leftrightarrow$ $\beta < \theta$

❑ Teorema de coseno

➢ **Caso 1**

$$cos(a) = cos(b) \cdot cos(c) + sen(b) \cdot sen(c) \cdot \boldsymbol{cos(\alpha)}$$

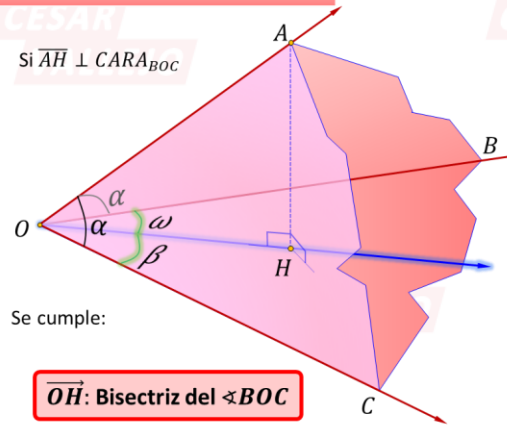
➢ **Caso 2**

$$cos(a) = -cos(\beta) \cdot cos(\theta) + sen(\beta) \cdot sen(\theta) \cdot \boldsymbol{cos(\alpha)}$$

TEOREMAS Particulares

Sea $O - ABC$ un triedro isósceles

Si $\overline{AH} \perp CARA_{BOC}$



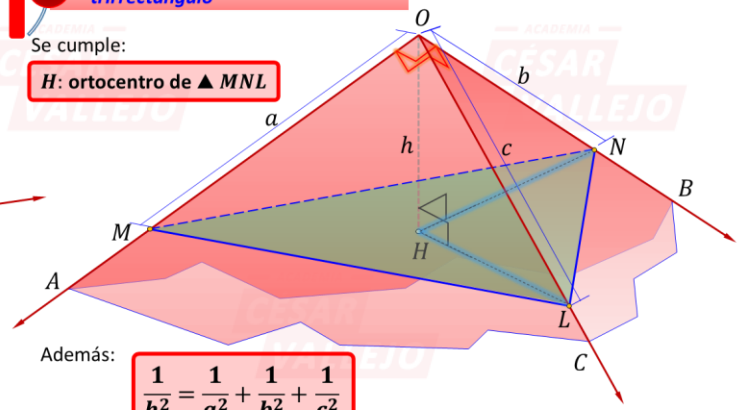
Se cumple:

$\overrightarrow{OH}$: Bisectriz del $\sphericalangle BOC$

$\rightarrow \beta = \omega$

Sea $O - ABC$ un triedro trirrectángulo

Si $\overline{OH} \perp \blacktriangle MNL$



Se cumple:

H: ortocentro de $\blacktriangle MNL$

Además:

$$\frac{1}{h^2} = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}$$

$$(\mathbb{A}_{NOL})^2 = (\mathbb{A}_{NHL})(\mathbb{A}_{NML})$$

$$(\mathbb{A}_{MNL})^2 = (\mathbb{A}_{MOL})^2 + (\mathbb{A}_{NOL})^2 + (\mathbb{A}_{MON})^2$$

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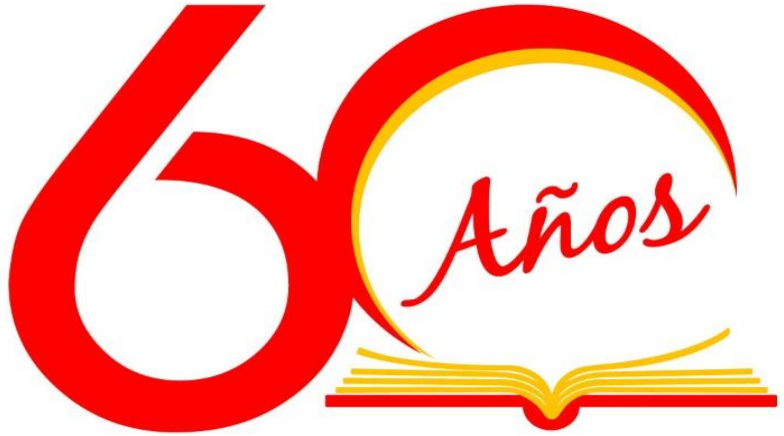

¡Gracias!